On some cancellation algorithms

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We define $b_f(n)$ to be the smallest positive integer d such that all the values $f(n_1, n_2, \ldots, n_m)$, where $n_1+n_2+\ldots+n_m \le n$ are not divisible by d. For the given functions $f : \mathbb{N}^m \to \mathbb{N}$, we will obtain the asymptotic characterisation of the sequence of the least non cancelled numbers $(b_f(n))_{n\in\mathbb{N}}$.

Browkin and Cao [BC] have shown that, in the case of the function $f : \mathbb{N}^2 \ni (n_1, n_2) \to n_1^2 + n_2^2 \in \mathbb{N}$, the sequence $(b_f(n))_{n\in\mathbb{N}}$ is the increasing sequence of all elements of the set of all square-free positive integers which are products of prime numbers $3 \pmod 4$. The aim of the talk is to present results of [TZI], [MZII] and [MZIII]. We investigate the functions:

$$f_1(n_1) = n_1^k,\ k \ge 2,\ f_2(n_1, n_2, \ldots, n_m) = n_1 n_2 \cdot \ldots \cdot n_m,\ m \ge 2,\ f_3(n_1, n_2, n_3) = n_1^2+n_2^2+n_3^2,$$

$$f_4(n_1, n_2, n_3, n_4) = n_1^2 + n_2^2 + n_3^2 + n_4^2,\ f_5(n_1, n_2) = n_1^j + n_2^j,\ j > 3,\ \text{odd}.$$

In the case $f : \mathbb{N}^2 \ni (n_1, n_2) \to {n_1}^3 + {n_2}^3 \in \mathbb{N}$, this characterisation can be reformulated in the terms of the permutation polynomials of the finite commutative quotient ring $\mathbb{Z}/m\mathbb{Z}$. There are situations in which we cannot expect explicit formula for $b_f(n)$ to be simple, but we can provide the upper and lower bounds of it.

[BC] J. Browkin, H-Q. Cao, *Modifications of the Eratosthenes sieve*, Colloq. Math. **135** (2014), 127-138.

[TZI] A. Tomski, M. Zakarczemny, *On some cancellation algorithms*, NNT-DMM **23** (2017), 101-114.

[MZII] M. Zakarczemny, *On some cancellation algorithms, II*, CzT to appear.

[MZIII] M. Zakarczemny, *On some cancellation algorithms, III*, to appear.